

# From Szeged number to closeness of networks

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Szeged number [2] is a topological index of molecular graphs studied in chemical graph theory. In fact, it is perhaps a more natural generalization of the original Wiener number defined on trees. The usual generalization, called the Wiener number of a graph, is just the sum of all distances. Needless to say, the Wiener number and its generalizations are of interest when considering the communication networks. Hence, the Hosoya polynomial [3], whose derivative at  $x = 1$  is the Wiener number, may be of general interest. We know that Hosoya polynomial can be computed in linear time on double weighted cactus graphs [4], likely also on some larger classes of networks. An example illustrating the use of Hosoya polynomial is its close relation to the recently defined measure of network centrality, namely the network closeness [1]. Based on the concept of closeness, decay stable graphs were defined in [5]. We prove one and discuss another conjecture of [5].

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## References

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