Additional pruning and backtracking rules in the Carradhan-Pardalos algorithm applied to packing by cubical clusters

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The problem of packing optimally a large cube by translated copies of a tripod can be reduced to a clique search problem. As a first step one constructs a suitable compatibility graph G. Then one feeds this graph G into a clique solver. In our case, we will use a version of the Caraghan–Pardalos algorithm. The procedure works with two subsets of the compatibility graph G. Namely, the clique under construction C and the set of prospective nodes P. One picks a vertex v of P and extends by adding v to C to get a larger clique and reduces P to the common neighbors of the elements of C. If P is empty then the search backtracks. One may anticipate backtracking before exhaustingly testing each element of P. We refer to this action as pruning of the search tree. The main result of this work is following. We define a directed graph D whose nodes are the vertices of the compatibility graph G. We show that if T is an optimal clique in G, then there is a clique T' such that the node set of T' induces a connected component in D. We can exploit D to speed up the Carraghan–Pardalos algorithm. If a vertex v in P is not an initial point of a directed edge of D whose terminal point is in $C \cup P$, then v can be deleted from P. If a vertex v in C is not an initial point of a directed edge of D with an end point in $C \cup P$, then we may backtrack. We carry out numerical experiments to test the practical utility of the suggested pruning and backtracking rules.

References

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