Robust coloring and related invariants of graphs

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A 1-selection f of a graph G is a function $f : V(G) \to E(G)$ such that $f(v)$ is incident to v for every vertex v. The 1-removed graph G_f has vertex set $V(G)$ and edge set $E(G) \setminus f(V(G))$. Motivated by applicability in extremal graph theory, we define the (1-)robust chromatic number $\chi_1(G)$ as the minimum of $\chi(G_f)$ over all 1-selections f of G. There is a natural analogous way to introduce the robust versions of many other graph invariants, too. We initiate a systematic study of this new area. Among various results, we compare the new parameters with the traditional ones, obtaining estimates that are tight in many cases. The definitions may also be extended to s-valued $V(G) \to E(G)$ mappings called s-selections, for any $s > 1$.