Shannon capacity and Hedetniemi-type equalities

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The categorical product of two graphs F and G, denoted $F \times G$, satisfies for all homomorphism monotone (increasing) graph parameters φ that $\varphi(F \times G) \leq \min\{\varphi(F), \varphi(G)\}$. Although it is famously not true for the chromatic number by the recent disproof of Hedetniemi's conjecture, some parameters give equality in this inequality. Examples of such parameters are the fractional chromatic number and the complementary Lovász number by results in [3] and [1], resectively. Both of these parameters are elements of what Zuiddam [4] calls the asymptotic spectrum of graphs. In the talk, that is based on [2], we elaborate on the consequences of the main result in [4] concerning the possibility of Shannon's graph capacity, the zero-error capacity of a noisy channel with distinguishability graph G, satisfying this Hedetniemi-type equality.

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References

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