Abstract for the SWORDS 2023 presentation:

Cliqueful graphs as a means of calculating the maximum number of maximum cliques of simple graphs

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Simple graphs on n vertices may contain a lot of maximum cliques (largest cliques in the graph) and maximal cliques (non-extendable cliques in the graph). But how many can they potentially contain? In 1965, Moon and Moser [1] have already calculated the maximum number of **maximal cliques** of simple graphs on n vertices. We show, that the maximum number of **maximum cliques** of simple graphs on n vertices is also the same number [2]:

$$M_n = \begin{cases} 3^{\lfloor n/3 \rfloor} \text{ if } n \bmod 3 = 0 \\ 4 \cdot 3^{(\lfloor n/3 \rfloor - 1)} \text{ if } n \bmod 3 = 1 \\ 2 \cdot 3^{\lfloor n/3 \rfloor} \text{ if } n \bmod 3 = 2 \end{cases}$$

For this, we introduce the family of so-called "cliqueful" graphs: graphs that can be fully determined by their set of maximum cliques. Then we prove, that this set definitely contains graphs with the maximum number of maximum cliques on n vertices $\mathrm{MG}(n)$. We will do this by further reducing the set that contains $\mathrm{MG}(n)$, to so-called "saturated cliqueful" graphs, and "composite saturated cliqueful" graphs. It will turn out that $\mathrm{MG}(n)$, if $n \geq 15$, is *almost* the same as composite saturated cliqueful graphs with prime components of size ≤ 4.

References

[1] Moon J.W., Moser L., On cliques in graphs, Israel journal of Mathematics, 3, 23–28 (1965)

[2] D. Pfeifer, Cliqueful graphs as a means of calculating the maximal number of maximum cliques of simple graphs (2023) (ArXiv 2307.14120)