Constructing bounded degree graphs with prescribed degree and neighbor degree sequences

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Let $D = \{d_i\}$ and $F = \{f_i\}$ be two sequences of positive integers. We consider the following decision problems: is there a i) multigraph, ii) loopless multigraph, iii) simple graph, iv) cycle-free graph (forest or tree), v) caterpillar $G = (V, E)$ such that for all k, $d(v_k) = d_k$ and $\sum_{w \in \mathcal{N}(v_k)} d(w) = f_k$ ($d(v)$ is the degree of v and $\mathcal{N}(v)$ is the set of neighbors of v). Here we show that all these decision problems can be solved in polynomial time if $\Delta := \max_k d_k$ is bounded. The problems are converted into an integer programming feasibility problem in which both the number of variables and the number of inequalities depend only on Δ but not on n. The problem is motivated by NMR spectroscopy of hydrocarbons. The algorithm has been implemented in the ZIMPL language, and its applicability is demonstrated on trees up to $n = 1000$ vertices. The average reconstruction time for trees with 1000 vertices is still less than 40 milliseconds.

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