Optimal Cutting Arrangements in 1D

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Given a warehouse stocked with steel rods of potentially varied lengths, we face the challenge of fulfilling incoming orders for rod pieces. The inherent cost associated with cutting these rods necessitates minimizing the number of cuts. A primary strategy we adopt is identifying exact fits – collections of orders that perfectly match and utilize the entirety of a warehouse rod. This approach not only matches the order but also conserves resources by eliminating the need for additional cuts to discard the "leftover rod segment."

In this presentation, we will explore both the theoretical and practical aspects of the above problem.

In the theoretical section, we will explore the relationship between the feasible solution and the optimal solution. Furthermore, we will prove an equivalent formulation of the problem, which will help us find a reduction of this problem to optimal bin-packing. This latter problem is known to be NP-complete; thus, no polynomial-time solution for our problem is currently known.

For the practical section, we will introduce two approaches. The first employs dynamic programming combined with clique search. The second approach uses a 0-1 integer programming formulation, which turns out to be much more efficient than the first method. Our implementation of the second approach, utilizing the Gurobi Solver, will be demonstrated using simulated data. A potential extension of the 0-1 integer programming using hierarchical optimization will also be discussed.

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