On strongly connected resolvable networks

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A Resolvable network (RN) is a data structure [1]. RNs are generalized digraphs. An RN contains subnetworks, a subnetwork contains nodes, and a node itself can be a subnetwork.

In an RN, subnetworks should not be distinct, they can overlap, but only distinct subnetworks can be connected by directed edges. The inner structure of subnetworks is unknown. Or in other words, we do not use the information on the inner structure of subnetworks to study RNs; but we use this information: how they are intersected by other subnetworks and connected to other subnetworks.

A reach consists of two connected subnetworks. A reach, say $A \to B$, can be represented by a clause, where nodes in A are represented by negative literals and nodes in B are represented by positive literals. A resolvable network may contain two special subnetworks: $Source$ and $Sink$. $Source$ has the property that there is no incoming edge to it. The $Sink$ has the property that there is no outgoing edge from it. The reach $Source \to B$ is represented by the clause, where nodes of B are represented by positive literals. The reach $A \to Sink$ is represented by the clause, where nodes of A are represented by negative literals. Any RN can be represented as a SAT problem. Furthermore, any SAT problem can be represented by an RN.

Since RNs are generalized digraphs, we can study the question: How to generalize notions of digraphs to the level of RNs? In this work, we generalize the notion of a strongly connected digraph. A digraph is strongly connected if and only if there is a path from any node to any other one [2]. It is not trivial to generalize this notion, since the notion of path is defined in the level of RNs. Therefore, we give a recursive list of RNs that are strongly connected. These examples have the following properties: A) Neither the *Source* nor the *Sink* is present. B) Their SAT representation has an equivalent SAT problem which contains only binary clauses, and each clause contains exactly one negative literal, and exactly one positive literal, and this SAT problem is represented by a strongly connected digraph. This second property ensures that the resulting RN will be a digraph, since each of its subnetworks are singleton sets, i.e., they contain only one node.

This definition is not very practical, because there is no practical algorithm that can generate an equivalent 2-SAT problem out of a general SAT problem. On the other hand, our recursive list of strongly connected RNs might help researchers to further analyze this problem.

References

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