Conway's Army Percolation

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We study a probabilistic version of the celebrated Conway's Army game. In this game the lower semiplane of an infinite chessboard is filled with checker pieces. A celebrated result due to Conway (1961) shows that one **cannot** move a checker piece in a finite number of steps to a position on row 5 (rows 1-4 are reachable).

In the version that we are interested in, inspired by percolation theory, we assign pieces to all cells of the lower semiplane independently by flipping a coin with success probability p (where p is a fixed constant between 0 and 1). We are interested in estimating $D_k(p)$, the probability that one can reach a fixed cell on line k, for $k = 1, 2, 3, 4$. We derive several results that provide lower and upper bounds on this probability. These results show that $0 < D_k(p) < 1$ for every $p \neq 0, 1$, and suggest the fact that the $D_k(p)$ is an analytical function of p. As a teaser for the talk, the results for $k = 4$ are summarized in the figure below. The green curve is a (direct) lower bound. The violet curve is a direct upper bound. The other curves are upper bounds based on tail bound inequalities: Markov's inequality (yellow), Cantelli's inequality (red), the Hoeffding inequality (blue). Despite visuals, the blue line is provably overtaken by the red line around $p = 0$ and by the violet line around $p = 1$.