Shannon capacity, Lovász theta number and the Mycielski construction

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The Shannon OR-capacity $C_{\mathrm{OR}}(G)$ of a graph G is defined as $C_{\mathrm{OR}}(G) = \lim_{t\to\infty} \sqrt[t]{\omega(G^t)}$ where G^t is an appropriately defined graph exponentiation (and ω stands for clique number). In [1] Lovász proved $C_{\mathrm{OR}}(C_5) = \sqrt{5}$ with the help of his famous ϑ-number introduced in [1]. The Mycielski construction is one of the standard constructions showing that a triangle-free graph can have arbitrarily large chromatic number. From a graph G it produces graph $M(G)$ having the same clique number while the chromatic number increases by 1. We investigate the effect of this construction on the complementary Lovász theta number $\overline{\vartheta}(G) = \vartheta(\overline{G})$ and on Shannon OR-capacity. For the former we prove that $\overline{\vartheta}(M(G))$ is determined by $\overline{\vartheta}(G)$ and give an explicit formula for it in terms of $\overline{\vartheta}(G)$. For Shannon OR-capacity we show that $C_{\mathrm{OR}}(M(G)) > C_{\mathrm{OR}}(G)$ whenever there exists a $k \in \mathbb{N}$ such that $C_{\mathrm{OR}}(G) = \sqrt[k]{\omega(G^k)}$.

The talk is based on the forthcoming paper [2].

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References

[1] László Lovász, On the Shannon capacity of a graph, *IEEE Trans. Inform. Theory* , 25, 1–7 (1979).

[2] Bence Csonka, Gábor Simonyi, Shannon capacity, Lovász theta number and Mycielsky construction, paper in preparation